Evaluation of Linearity by Transformation of the Amplitude Characteristic

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Abstract—**This paper addresses the problem of defining, measuring, and evaluating linearity of memoryless, nonlinear devices with relatively narrow bandwidths. The analysis is based on the single-carrier transfer characteristic and leads to a definition of linearity in terms of a transform which is determined by the input-signal composition and the working conditions of the device.**

I. INTRODUCTION

A BASIC PROBLEM in engineering is that of translating system requirements to a set of measurable parameters of individual elements. Some of these parameters may have a simple physical meaning. Others may not even be well defined. One such parameter which is of great importance for most amplifying and frequency-translating equipment is *linearity*.

Linearity is usually measured and expressed in terms of intermodulation product levels for very simple input signals, e.g., two carriers, from which the systems designer must estimate how a nonlinear device will perform in a particular working condition.

This paper defines transforming functions which can be used to characterize device nonlinearities. The functions are based on a recently developed technique for analysis of nonlinear distortion [2]–[4], and the paper addresses the following problems.

1) The definition of suitable measurements to find parameters from which the nonlinear distortion can be calculated.

2) The definition of transforming functions.

3) The computational technique to find signal and intermodulation noise levels.

The results are directly applicable to memoryless devices whose output envelope and phase is determined by the instantaneous input envelope, but the method can also be extended to frequency-translating and frequency-multiplying devices.

II. BASIC DATA FOR ANALYSIS

The "classical" methods for analysis of nonlinear distortion [1] are based on a knowledge of the voltage transfer function (dc out versus dc in), $v(u)$. This can usually not be measured directly on narrow-band devices like microwave tubes, but Blachman [5] has shown that the odd part of $v(u)$ is uniquely determined by $v_1(a)$ which relates the amplitude of the fundamental output component to the amplitude of a single input sine wave. Shimbo [2] has derived a general expression for the level of any spectral component in the first zone of the output of a nonlinear bandpass device excited by a number of (phase-modulated) sine waves plus Gaussian noise. Reference [3, eq. (15)] gives Shimbo's expression of the following form

$$C_{\{p\}} = \int_0^\infty a v_1(a)\, da \int_0^\infty x J_1(ax) \prod_{n=1}^{N} J_{p_n}(A_n x) \cdot \exp\left[-(P/2)x^2\right] dx \quad (1)$$

where $C_{\{p\}}$ is the amplitude of a particular output component characterized by $\{p\} = p_1 p_2\ p_N$, A_1 to A_N are the amplitudes of the N sine waves at the input, and P is the power of the stationary Gaussian input noise. J_1 and J_{p_n} are Bessel functions of the first kind and of order 1 and p_n, respectively. It is significant to note that (1) can be expressed as the integral of the product of two functions, one determined only by the nonlinear characteristic and the other only by the input-signal composition [4]. The expressions can also be extended to nonlinear devices with amplitude-to-phase (AM–PM) conversion by expressing $v_1(a)$ as a complex function

$$\boldsymbol{v}_1(a) = |v_1(a)| \exp\left[j\zeta(a)\right] \quad (2)$$

where $\zeta(a)$ is phase shift versus input amplitude (envelope).

It has been demonstrated [4] that good agreement can be achieved between the levels of measured intermodulation products and the levels calculated from the complex amplitude characteristic $\boldsymbol{v}_1(a)$ (two-tone measurements with up to fifth-order intermodulation products). The complex $\boldsymbol{v}_1(a)$ represents, therefore, useful basic data for any linearity analysis, in particular for more severe nonlinearities where measurement accuries in determining the departure from linearity are less critical. The importance of AM–PM conversion must be emphasized as illustrated in Fig. 1 which shows the calculated signal level to third-order IM product-level ratio, C_{10}/C_{21}, versus useful output power for a traveling-wave tube amplifier (TWTA) excited by two equal-level sinusoidal carriers. The calculations were performed for three different helix overvoltages, 0, 3, and 6 percent. An analysis based entirely on the amplitude characteristic would lead to the wrong

Manuscript received November 20, 1973; revised November 11, 1974.

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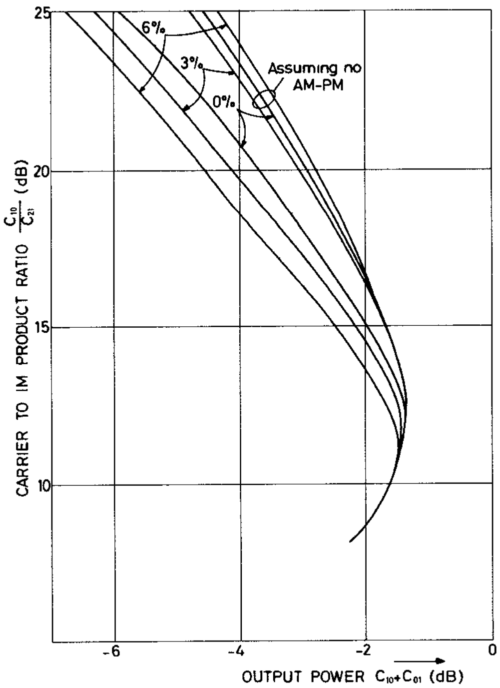

Fig. 1. Carrier-to-IM ratio versus output power for different overvoltages, 0, 3, and 6 percent, calculated with and without AM–PM conversion.

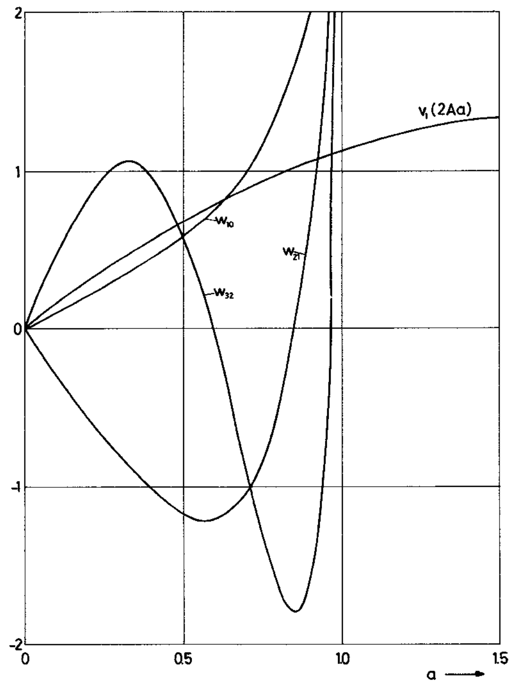

Fig. 2. Transforming functions for two sine waves of equal level and first-zone IM components up to order 5.

conclusion that increasing overvoltage improved the linearity. The AM–PM conversion increased with overvoltage and when $\zeta(a)$ was properly included the analysis showed a reduction of linearity with increasing value and this was in conformity with measured results.

III. CHOICE OF TRANSFORMING FUNCTION

Equation (1) shows that any output component $C_{\{p\}}$ in the first zone can be obtained by multiplying $v_1(a)$ by a particular function $w_{\{p\}}$ and integrating over the possible values that the input envelope can assume, in general 0 to ∞.

$$C_{\{p\}} = \int_0^\infty v_1(a) w_{\{p\}} \, da \tag{3}$$

where

$$w_{\{p\}} = a \int_0^\infty x J_1(ax) \prod_{n=1}^{N} J_{p_n}(A_n x) \exp\left[-(P/2)x^2\right] dx. \tag{4}$$

This leads to a natural definition of linearity in terms of a system of transforms which are applied to the device under examination. The choice of transforming functions, however, must be based on system parameters like input-signal composition and spectrum and on the effect of given noise components on system performance.

One obvious property of the transforms corresponding to intermodulation products is that when applied to a straight line they must all give zero. Likewise, the transform corresponding to signal component number k, C_k, must for the same integration give A_k independent of input-noise power P and the level of the other input carriers. These properties can be used to check the accuracy of the numerical operations.

As an illustration, Fig. 2 shows the set of transforming functions corresponding to carrier and first-zone intermodulation products of order 3 and 5 when the input signal consists of two sinusoids of equal amplitudes A [4]. Fig. 2 also shows a typical saturation characteristic $v_1(2Aa)$ where the factor $2A$ in the argument is due to the normalization of the w-functions to the interval $[0,1\rangle$.

It is now possible to calculate a particular set of transforms which can be used for a relastic evaluation and performance analysis of nonlinear devices for given operating conditions. They are determined by the input-signal composition (amplitude distribution) according to (4) and by the input spectrum, according to the effect on system performance of various IM components, but they are independent of the shape of the nonlinearities involved.

As an example, in a satellite system with a transponder carrying several carriers simultaneously there could be reasons for evaluating the linearity of the ground-terminal transmitter and the transponder by different sets of transforms. The ground terminals would normally carry a lower number of signals. For the evaluation of the transponder it may be sufficient to consider the input signal as having a Gaussian amplitude distribution. According to a theorem by Middelton [6] the total amount of intermodulation noise power is independant of the shape of the input power spectrum. The total noise-power density at any particular frequency is therefore given by the weighted sums of the third-, fifth-, etc., order spectra obtained by repeated convolutions of the input spectrum. Only the *weights* depend on the shape of the nonlinearity.

Reference [4, eq. (7)] gives an expression for the "useful" output signal P_1 and the power of the first few intermodulation spectra P_3, P_5, etc., when the total input signal to the device can be considered as Gaussian. A general criterion for evaluation of linearity could be

$$R = (P_3/3^{1/2} + P_5/5^{1/2} + \cdots)/P_1$$

where the square-root terms are included to account for the increased spectral spread of higher order components.

IV. ORGANIZATION OF COMPUTATIONS

Equations (3) and (4) can form the basis for the numerical analysis of measured characteristics. When the total input signal is Gaussian or when a Gaussian term is dominant it is convenient to normalize the coefficient of the quadratic exponential term to unity and to apply Hermite integration [7].

When the input signal consists of sinusoids only, it is convenient to normalize (3) and (4) by introducing the sum S of the input amplitudes. The infinite upper limit of integral (3) can then be replaced by unity since the output does not depend upon $v_1(a)$ for $a > S$. Equation (4) needs only to be computed once for each signal *composition*

$$v_{\{p\}} = \int_0^1 v_1(Sa)w_{\{p\}}(a)\, da \tag{5}$$

$$w_{\{p\}} = a\int_0^\infty xJ_1(ax) \prod_{n=1}^{N} J_{p_n}[(A_n/S)x]\, dx. \tag{6}$$

The Gaussian quadrature formula [6] has been found useful for general application and particularly efficient computation can be achieved when the weight functions are tabulated for those values of the argument which are used for the numerical integration.

The integral (6) causes numerical problems when the number of carriers is small. This can be ameliorated by replacing $xJ_1(ax)$ by $-(\partial/\partial a)J_0(ax)$ and integrating by parts. Thus (5) and (6) become

$$v_{\{p\}} = \int_0^1 (1/a)(\partial/\partial a)[av_1(a)]Z_{\{p\}}(a)\, da \tag{7}$$

where

$$Z_{\{p\}} = a\int_0^\infty J_0(ax) \prod_{n=1}^{N} J_{p_n}[(A_n/S)x]\, dx. \tag{8}$$

Expression (8) does not have the factor x under the integral. The function $[av_1(a)]$ can be obtained from $v_1(a)$ by numerical differentiation, but it can also be measured directly since it is twice the gain seen by an infinitesimal sinusoid sharing the nonlinear device with a larger carrier of amplitude a as shown in the Appendix.

V. CONCLUSION

The linearity of a memoryless device can be evaluated for a particular application by a set of integral transforms of the, in general complex, single-carrier amplitude characteristic. The transforming functions are determined by the input-signal composition (amplitude distribution), and by the deteriorating effect that various intermodulation components will have on the system where the device will be employed. The functions are, however, independent of the shape of the nonlinearity.

APPENDIX
THE SMALL-SIGNAL GAIN OF NONLINEAR DEVICES

This appendix derives an expression for the effective gain seen by an infinitesimal sinusoid accessing a nonlinear device in the presence of a sinusoid with given amplitude A. The nonlinear device, which is assumed to be memoryless, is characterized by its single-carrier amplitude characteristic $v_1(a)$. The gain g can be expressed as

$$g = \lim_{\epsilon\to 0} (1/\epsilon) \int_0^\infty av_1(a)\, da \int_0^\infty xJ_1(ax)J_1(\epsilon x)J_0(Ax)\, dx. \tag{A1}$$

The second integral is zero for $a < A - \epsilon$ and for $a > A + \epsilon$. Replacing $xJ_1(ax)$ by $-(\partial/\partial a)J_0(ax)$ and integrating by parts will therefore give only one term, provided that $A > \epsilon$.

$$\begin{aligned} g &= -\lim_{\epsilon\to 0} (1/\epsilon) \int_0^\infty av_1(a)\, da(\partial/\partial a) \\ &\quad \cdot \int_0^\infty J_0(ax)J_1(\epsilon x)J_0(Ax)\, dx \\ &= \int_0^\infty (\partial/\partial a)[av_1(a)]\, da \cdot \lim_{\epsilon\to 0} (1/\epsilon) \\ &\quad \cdot \int_0^\infty J_0(ax)J_1(\epsilon x)J_0(Ax)\, dx. \end{aligned} \tag{A2}$$

The second integral of (A2) can be solved analytically

$$\int_0^\infty J_0(ax)J_1(\epsilon x)J_0(Ax)\, dx = \begin{cases} \dfrac{1}{\pi\epsilon} \arccos\left(\dfrac{a^2 + A^2 - \epsilon^2}{2aA}\right), & \text{when } A,\ \epsilon, \text{ and } a \text{ form the sides of a triangle} \\ 0, & \text{otherwise.} \end{cases}$$

The integral is nonzero for a in the interval $\langle A - \epsilon, A + \epsilon\rangle$, where it is positive for all values of a. The area under the curve is given by

$$\begin{aligned} S &= \int_{A-\epsilon}^{A+\epsilon} \frac{1}{\pi\epsilon} \arccos\left(\frac{a^2 + A^2 - \epsilon^2}{2aA}\right) da \\ &= \frac{1}{\pi\epsilon} \int_{-\epsilon}^{\epsilon} \arccos\left(1 + \frac{y^2 - \epsilon^2}{2A^2 + 2yA}\right) dy \end{aligned}$$

where $y = a - A$.

When $\epsilon \ll A$, $2A^2 + 2yA \approx 2A^2$. The arccosine function can be replaced by arcsine and only the first term of the series expansion needs to be included.

$$S = \frac{1}{\pi\epsilon}\int_{-\epsilon}^{\epsilon} \arcsin\left[\frac{\epsilon^2 - y^2}{A^2} - \frac{1}{4}\left(\frac{\epsilon^2 - y^2}{A^2}\right)^2\right]^{1/2} dy$$

$$\approx \frac{1}{\pi\epsilon}\int_{-\epsilon}^{\epsilon} \arcsin\left(\frac{\epsilon^2 - y^2}{A^2}\right)^{1/2} dy$$

$$\approx \frac{1}{\pi\epsilon A}\int_{-\epsilon}^{\epsilon} (\epsilon^2 - y^2)^{1/2}\, dy = \frac{1}{\pi\epsilon A}\cdot\tfrac{1}{2}\pi\epsilon^2 = \frac{\epsilon}{2A}.$$

Consequently,

$$\lim_{\epsilon\to 0} \frac{2(Aa)^{1/2}}{\epsilon}\int_0^{\infty} J_1(\epsilon x)J_0(ax)J_0(Ax)\, dx = \delta(A - a) \tag{A3}$$

and

$$g = \frac{1}{2A}\frac{\partial}{\partial A}[Av_1(A)] = \frac{1}{2}\left(v_1'(A) + \frac{v_1(A)}{A}\right). \tag{A4}$$

Therefore, the gain seen by the smaller carrier is given by the arithmetic mean of the slope of $v_1(a)$ and the large-signal gain $v_1(a)/a$ for $a = A$.

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A Dynamic Calibration Method for Biphase Phase-Shift-Keyed Modulators

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***Abstract*—A method is proposed whereby a (microwave) phase bridge can be constructed to *simultaneously* measure both the phase and amplitude balance of a biphase phase-shift-keyed (PSK) modulator. Given an initial *narrow-band* phase shifter that is capable of either continuous uncalibrated phase shifting or slow switching between calibrated fixed-90° phase shifts, the phase and amplitude balance of a second wide-band phase shifter (biphase modulator) can be determined. Furthermore, these measurements may be dynamically and simultaneously displayed in real time on a dual-trace oscilloscope. A significant feature of the method is the unique identification of either or both the phase and amplitude unbalance by means of the symmetric or asymmetric features that these unbalances induce in the display pattern.**

Quantitative criteria for the sensitivity of the technique are presented.

I. INTRODUCTION

THIS PAPER describes a scheme whereby a phase-bridge configuration can be used to characterize a RF phase-shift-keyed (PSK) biphase modulator for simultaneous phase and amplitude balance under dynamic conditions. The technique is applicable at any frequency where the basic bridge components are realizable, although the terminology used here is sometimes suggestive of waveguide.

Following this introductory section, Section II develops the two equations which describe the detected bridge voltages in terms of the modulator's phase and amplitude balance. Section II is therefore concerned with the appropriate representations for the phase bridge and the essential properties of components from which it is constructed. Given these results, Section III describes the waveforms that result when these voltages are displayed on a dual-trace oscilloscope. Five examples are described in detail with corresponding figures. The first example depicts the patterns for a perfectly balanced PSK modulator. The four remaining examples depict the patterns of special cases when the PSK modulator is slightly unblanced. Use of these latter four figures permits *any combination* of amplitude and phase unbalance of a PSK modulator to be readily identified in a quantitative manner. Section IV summarizes the salient features of the material developed in this paper.

Manuscript received December 1, 1973; revised June 1, 1974.
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